Exercise 49

Find the limit.

$$\lim_{x\to\pi/4}\frac{1-\tan x}{\sin x-\cos x}$$

Solution

Rewrite the limit.

$$\begin{aligned}\lim_{x\to\pi/4}\frac{1-\tan x}{\sin x-\cos x}=\lim_{x\to\pi/4}\frac{1}{\cos x}\cdot\frac{1-\tan x}{\frac{\sin x}{\cos x}-1}&=\lim_{x\to\pi/4}\frac{1}{\cos x}\cdot\frac{1-\tan x}{\tan x-1}\\&=\lim_{x\to\pi/4}\frac{-1}{\cos x}\cdot\frac{1-\tan x}{1-\tan x}\\&=\lim_{x\to\pi/4}\frac{-1}{\cos x}\\&=-\frac{1}{\cos\frac{\pi}{4}}\\&=-\sqrt{2}\end{aligned}$$